Int J Thermophys (2018) 39:17
https://doi.org/10.1007/s10765-017-2336-1

# A Two-Temperature Photothermal Interaction in a Semiconductor Medium Containing a Cylindrical Hole

**Ibrahim A. Abbas**[1,2] · **Aatef Hobiny**[2]

Received: 7 December 2016 / Accepted: 13 November 2017
© Springer Science+Business Media, LLC, part of Springer Nature 2017

**Abstract** Photothermoelastic interactions in an infinite semiconductor medium containing a cylindrical hole with two temperatures are studied using mathematical method under the purview of the coupled theory of thermal, plasma and elastic waves. The internal surface of the hole is constrained and the carrier density is photogenerated by bound heat flux with an exponentially decaying pulse. Based on Laplace transform and the eigenvalue approach methodology, the solutions of all variables have been obtained analytically. The numerical computations for silicon-like semiconductor material have been obtained. The results further show that the analytical scheme can overcome mathematical problems to analyze these problems.

**Keywords** A semiconducting material · Cylindrical hole · Eigenvalue approach · Laplace transform · Two-temperature

## 1 Introduction

The development of spatially resolved in situ probes for the investigation of transport phenomena in solids has attracted much attention. In the present work of research, we try to measure transport processes based on the principle of optical beam deflection through a photothermal approach that can be considered as an expansion of the

✉ Ibrahim A. Abbas
ibrabbas7@yahoo.com

Aatef Hobiny
ahobany@kau.edu.sa

1 Department of Mathematics, Faculty of Science, Sohag University, Sohâg, Egypt

2 Nonlinear Analysis and Applied Mathematics Research Group (NAAM), Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia

Published online: 27 November 2017

photothermal deflection technique. Such a technique is characterized by being contactless and it directly yields the parameters of the electronic and thermal transport at the semiconductor surface or at the interface and within the bulk of a semiconductor. Pure silicon is intrinsic semiconducting and it is used in a wide range of semiconducting industries; for example, the monocrystalline Si is used to produce silicon wafers.

The thermoelasticity theory with two-temperature was established by Gurtin and Williams [1], Chen et al. [2] and Chen and Gurtin [3], wherein by using another one depending on two temperatures (thermodynamic temperature $T^*$ and conductivity temperature $\phi^*$), the classical Clausius–Duhem inequality has been replaced: The first is due to the inherent mechanical processes between the particles and the elastic material layers, and the second is due to the thermal processes. Over the last decade, the theory of two-temperature thermoelasticity has been observed, developed in numerous works, and finds applications mainly in problems in which the constraints of the discontinuities do not involve a physical interpretation. Carrera et al. [4] studied the effect of two-temperature theory in the vibrational analysis for an axially moving microbeam. Abbas et al. [5] investigated the response of thermal source without energy dissipation with two temperatures in transversely isotropic thermoelastic materials. Deswal and Kalkal [6] studied the effects of initial stress with two-temperature magneto-thermoelasticity using the state space formulation.

The various effects of thermoelastic and electronic deformation in semiconducting medium with neglecting the coupled system of thermoelastic and plasma equations were analyzed by many authors [7–9]. Previously, plasma field facilities, heat and micromechanical in one dimension were analyzed experimentally and theoretically as in Todorovic et al. [10–12]. In this study, a theoretical analysis to describe these two phenomena that give information about the properties of carrier recombination and transport in the semiconductor is provided. The changes in the propagation of plasma and thermal waves due to the linear coupling between heat and mass transport (i.e., thermos diffusion) were included. Opsal and Rosencwaig [13] presented their studying of semiconducting material based on the results shown by Rosencwaig et al. [14]. Abbas [15] studied a dual phase lag model on photothermal interaction in an unbounded semiconductor medium with a cylindrical cavity. Hobiny and Abbas [16] investigated the photothermal waves in an infinite semiconducting medium with a cylindrical cavity. In the domain of Laplace, the approach of eigenvalue gives an exact solution without any restrictions on the actual physical quantity assumptions.

In the present work, based on the coupled theory of thermoelastic and plasma wave with two temperature, the photothermoelastic interaction in an infinite semiconducting material containing a cylindrical hole is studied. By using the eigenvalues approach and Laplace transform, the governing equations are processed using an analytical–numerical technique. Numerical calculations were made for silicon-like semiconductor medium, and the results have been verified numerically and are represented graphically to show the effect of two-temperature parameter on the physical quantities.

## 2 Physical Model and Mathematical Formulation

The theoretical analysis of the transport processes in a semiconductor material is provided along with studying coupled elastic, thermal and plasma waves simultaneously. A homogeneous semiconducting material is considered. The key physical quantities are the distribution of temperature $T(\boldsymbol{r}, t)$, the density of carriers $n(\boldsymbol{r}, t)$ and the elastic displacement components $u_i(\boldsymbol{r}, t)$. For an isotropic, elastic and homogeneous semiconductor, the governing equations of motion, plasma and heat conduction under the two-temperature photothermal theory can be described as [17,18]:

$$\mu u_{i,jj} + (\lambda + \mu)\, u_{j,ij} - \gamma_n N_{,i} - \gamma_t T_{,i} = \rho \frac{\partial^2 u_i}{\partial t^2}. \tag{1}$$

$$D_e N_{,jj} = \frac{\partial N}{\partial t} + \frac{N}{\tau} - \frac{k}{\tau} T, \tag{2}$$

$$K\phi_{,jj} = \rho c_e \frac{\partial T}{\partial t} - \frac{E_g}{\tau} N + \gamma_t T_o \frac{\partial u_{j,j}}{\partial t}. \tag{3}$$

The heat conduction is correlated with the dynamical heat through the expression [2,3]

$$T = \phi - b\phi_{,jj}. \tag{4}$$

The constitutive relationship was written in the form:

$$\sigma_{ij} = \mu \left(u_{i,j} + u_{j,i}\right) + \left(\lambda u_{k,k} - \gamma_n N - \gamma_t T\right) \delta_{ij}, \tag{5}$$

where $\lambda, \mu$ are the Lame's constants, $N = n - n_o$, $n_o$ is the carrier concentration at equilibrium, $T_o$ is the reference temperature, $\phi = \phi^* - T_o$, $\varphi^*$ is the increment of temperature conductivity, $T = T^* - T_o$, $T^*$ is the increment of thermodynamic temperature, $b > 0$ is the two-temperature parameter, $\rho$ is the medium density, $u_i$ are the displacement components, $\sigma_{ij}$ are the stress components, $c_e$ is the specific heat at constant strain, $\gamma_n = (3\lambda + 2\mu)d_n$, $d_n$ is the electronic deformation coefficient, $\gamma_t = (3\lambda + 2\mu)\alpha_t$, $\alpha_t$ is the linear thermal expansion coefficient, $\boldsymbol{r}$ is the position vector, $K$ is the thermal conductivity, $\tau$ is the photogenerated carrier lifetime, $t$ is the time, $D_e$ is the coefficient of carrier diffusion, $k = \frac{\partial n_o}{\partial T}$ is the coupling parameter of thermal activation [18] and $i, j, k = r, \theta, z$ for cylindrical coordinates. Let us consider a homogeneous isotropic infinite semiconducting medium containing a cylindrical hole, whose state can be expressed in terms of the space variable $r$ and the time $t$ which occupies the region $a \le r < \infty$. The cylindrical coordinates $(r, \theta, z)$ are taken with $z$-axis aligned along the cylinder axis. Due to symmetry, only the radial displacement $u_r = u(r, t)$ is non-vanishing; therefore, Eqs. 1–5 may be expressed by:

$$(\lambda + 2\mu)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \gamma_n \frac{\partial N}{\partial r} - \gamma_t \frac{\partial \mathrm{T}}{\partial r} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{6}$$

$$D_e \left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r}\frac{\partial N}{\partial r}\right) = \frac{\partial N}{\partial t} + \frac{N}{\tau} - \frac{k}{\tau} T, \tag{7}$$